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The nuclear spin relaxation rate for clean quasi-two-dimensional superconductors in the vicinity of H_{c_2}

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Abstract. We calculate the nuclear spin relaxation rate of clean quasi-two-dimensional (2D) superconductors with s-wave pairing in high magnetic fields using the BCS model and the Brandt, Pesch and Tewordt approximation for the single-particle propagator. The calculation is valid in the vicinity of $H_{c_2}(T)$. Thermal fluctuations of vortices are not taken into account. We evaluate numerically our results in the case of high-T_c materials and organic superconductors. In a perpendicular field the Hebel–Slichter coherence peak is suppressed depending on the variation of the quasi-particle lifetime below T_c. We also describe the magnetic field dependence of the nuclear spin relaxation rate. Using the experimentally determined value of the quasi-particle lifetime, the model reproduces the experimental initial variations of T_1^{-1} versus T/T_c at different magnetic fields. However, the Hebel–Slichter peak is conserved in the low-field limit, or when the applied magnetic field is parallel to the superconducting planes.

1. Introduction

We present a calculation of the temperature dependence below T_c of the nuclear spin relaxation rate of quasi-2D superconductors in the clean limit ($l \gg \xi_a(0)$, where l is the electron mean free path and $\xi_a(0)$ is the coherence length in the a–b plane) and high-field regime (H close to H_{c_2}) within the framework of BCS theory [1]. Then we compare the theoretical results with experimental data obtained in high-T_c compounds and organic superconductors; these materials are quasi-2D systems in the clean limit. Our calculation is based on a perturbation expansion; it is valid in the vicinity of H_{c_2}.

The nuclear relaxation rate T_1^{-1} arises from the presence of direct and indirect coupling between the nuclear spins system and its surroundings, i.e. the electronic spins. This coupling leads to transitions between nuclear spin state levels and results in a lifetime broadening of the resonance line and relaxation to equilibrium with the electronic spin system. Measurements of T_1^{-1} provide valuable information on the mixed state of type II superconductors, particularly the nature of particle excitations since the transition rates are dependent on the strength of the spin–lattice coupling as well as the effective number of available states, and thus the coherent occupation of the superconducting condensed state. In the framework of the BCS theory and in the low-field regime ($H \ll H_{c_1}$) the dependence of T_1^{-1} on the density of state leads to a divergence near T_c which results in the well known Hebel–Slichter coherence peak in the variation of T_1^{-1} versus the temperature [2].

One striking feature of high-T_c materials and organic superconducting compounds is the absence of the coherence peak in the nuclear spin relaxation rate T_1^{-1} [3–11]. This

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property has been a central argument in theories for high-T_c materials based on either weak coupling or strong coupling [12] and has in this regard served as a test for the validity of the various theoretical models. However, most theoretical studies of T_1^{-1} in high-T_c superconductors have been conducted in the low-field limit ($H \ll H_{c_1}$) while experimental measurements have been mainly performed in strong magnetic fields ($H \gg H_{c_1}$). In the strong magnetic field regime, the nuclear spin relaxation rate shows not only the absence of the coherence peak, but also a magnetic field dependence in both high-T_c superconductors and superconducting organic materials [9, 11]. The absence of the Hebel–Slichter anomaly is independent of the field orientation. A study of T_1^{-1} in clean quasi-2D superconductors at large fields is therefore of interest.

An early theoretical analysis of the nuclear spin relaxation rate in bulk superconducting systems in the dirty limit (i.e. $l \ll \xi_a(0)$) with an applied magnetic field H comparable to $H_{c_2}(T)$ was carried out by Cyrot [13]. For this regime, he found that when $T \ll T_c(0)$, where T is the temperature and $T_c(0)$ the transition temperature in zero magnetic field, T_1^{-1}/T_{1N}^{-1} is smaller than unity, and when $T \sim T_c(0)$, T_1^{-1}/T_{1N}^{-1} is larger than unity. T_{1N}^{-1} is the nuclear spin relaxation rate in the normal state. This result which has been confirmed experimentally [14] is however not appropriate for clean superconductors among which a prominent class is provided by high-T_c materials and organic superconductors. A theory of NMR properties of clean three-dimensional (3D) superconductors has been investigated earlier [15] using perturbation expansions in powers of the order parameter which break down for small frequency ω. A more refined theory of the nuclear spin relaxation rate in clean 3D superconducting systems under strong magnetic field was worked out by Brandt and Pesch in a manner which extended the regime of validity of previous work [15, 16]. However, the anisotropic structure of high-T_c materials and organic superconductors indicates that theoretical investigations should address the quasi-2D structure of these materials. Although the conventional BCS theory (i.e. weak electron–phonon coupling, s-wave order parameter) is generally not believed to describe superconducting cuprates, it is of interest to know in detail its content for quasi-2D superconductors in the clean limit under strong magnetic fields. This is our aim in this paper.

Making use of the single particle propagator derived by Brandt, Pesch and Tewordt (BPT) [17], we study the nuclear spin relaxation rate of clean 2D superconductors in a strong magnetic field within the BCS model. We find that the Hebel–Slichter coherence peak is suppressed in a perpendicular field depending on the quasi-particle relaxation rate below T_c. This suppression of the Hebel–Slichter peak is in agreement with experiments on cuprates and organic superconductors. However, in contradiction to experiments, the anomaly is restored at low fields. Furthermore, the anomaly is also restored if the external field is rotated so as to become parallel to the *a*–*b* plane, which is also in contradiction with experiments. This is not surprising given the inadequacy of the simple BCS theory to describe high-T_c materials and organic superconductors. The theory outlined in this paper is expected to describe in a more satisfactory way clean quasi-2D materials when superconductivity is due to a weak electron–phonon coupling mechanism. Nonetheless, our results may suggest that the field dependence of T_1^{-1} observed in high-T_c materials and organic superconductors in the strong magnetic field regime is linked with the quasi-2D structure. This work deals with the mixed phase close to H_{c_2}; in that part of the phase diagram, vortex lines in strongly anisotropic superconductors are insensitive to pinning centres; they are in a liquid state [18]. Our calculation takes no account of the liquid state thermal fluctuations, and concentrates on the effect of the order parameter fluctuations. We show that the latter appear to be the dominant factor in the behaviour of the nuclear relaxation time close to H_{c_2}.

2. Nuclear spin relaxation rate in clean 2D superconductors

Brandt, Pesch and Tewordt (BPT) have in an early work studied clean 3D superconductors in high magnetic fields [17] (see appendix). Assuming a rigid Abrikosov flux lines lattice and an s symmetry order parameter, these authors derived the expression for the single-particle propagator for clean type II superconductors in strong magnetic fields in the case where the space variation of the order parameter Δ is neglected by disregarding the small components for $k \neq 0$ with respect to that at $k = 0$ in the Fourier-series development of the order parameter. This approximation becomes very good in the London limit, i.e. when $\kappa \gg 1$ where $\kappa = \lambda(0)/\xi(0)$. $\lambda(0)$ and $\xi(0)$ are the penetration length and the coherence length, respectively. We will suppose that this limit is relevant for our system. The nuclear spin–lattice relaxation rate T_1^{-1} is given by

$$\frac{1}{T_1 T} \sim \frac{1}{\omega_0} \operatorname{Im} \chi(\omega_0)$$

where $\chi(\omega_0)$ is the dynamic local response and ω_0 the Larmor nuclear frequency. T is the temperature.

Following Houghton and Maki, the imaginary part of the spin susceptibility in the low-frequency limit for clean superconducting systems can be written in terms of the BPT Green functions as [19]

$$\lim_{\omega\to 0} \frac{\operatorname{Im} \chi(\omega)}{\omega} = C \int \frac{\mathrm{d}E}{2T} \operatorname{sech}^2\left(\frac{E}{2T}\right) [N^2(E) + M^2(E)] \tag{1}$$

where C is a constant,

$$N^2(E) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{\mathrm{d}^3 p'}{(2\pi)^3} \operatorname{Im} G(\xi, E + \mathrm{i}\delta) \operatorname{Im} G(\xi', E + \mathrm{i}\delta) \tag{2}$$

$$M^2(E) = \Delta^2 \int \mathrm{d}u \rho(u) \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{\mathrm{d}^3 p'}{(2\pi)^3} \operatorname{Im} F(\xi, u, E - \mathrm{i}\delta) \operatorname{Im} F^+(\xi', u, E + \mathrm{i}\delta) \tag{3}$$

$$\rho(u) = \left(\varepsilon\sqrt{\pi}\right)^{-1} \mathrm{e}^{-(u/\varepsilon)^2} \qquad \varepsilon = k v_F \sin\theta \qquad \xi = v(p - p_F).$$

Here $k = (2eH/\hbar)^{1/2}$ is the reciprocal lattice vector of the flux-line lattice and v_F is the Fermi velocity. θ is the angle between the particle momentum and the applied magnetic field.

The BPT normal Green function is given by

$$G^{-1}(\xi, \omega + \mathrm{i}\delta) = \omega + \mathrm{i}\delta - \xi - \Delta^2 \int_{-\infty}^{+\infty} \mathrm{d}u \frac{\rho(u)}{\omega + \mathrm{i}\delta + \xi - u} \tag{4}$$

and the anomalous Green function F is written as

$$F(\xi, u, \omega + \mathrm{i}\delta) = G(\xi, \omega + \mathrm{i}\delta)[\omega + \mathrm{i}\delta + \xi - u]^{-1}. \tag{5}$$

For an anisotropic superconductor made up of superconducting planes stacked along an axis taken as the c-axis, in the configuration where the applied magnetic field is perpendicular to the planes, $\sin\theta$ is equal to unity and we can rewrite equations (2) and (3) as follows:

$$N^2(E) = \left(\frac{2\pi}{d}\right)^2 \left(\frac{p_F}{v_F (2\pi)^2}\right)^2 \left[\int \mathrm{d}\xi \operatorname{Im} G(\xi, E + \mathrm{i}\delta)\right]^2 \tag{6}$$

$$M^2(E) = \Delta^2 \left(\frac{2\pi}{d}\right)^2 \left(\frac{p_F}{v_F (2\pi)^2}\right)^2 \int \mathrm{d}u \rho(u) \int \mathrm{d}\xi\, \mathrm{d}\xi' \operatorname{Im} F(\xi, u, E - \mathrm{i}\delta) \times \operatorname{Im} F^+(\xi', u, E + \mathrm{i}\delta) \tag{7}$$

where d is the interlayer distance.

Carrying out the integration in equation (6), we obtain a term proportional to the square of the BPT density of state for quasi-2D systems:

$$N^2(E) = \pi^2 N_0^2 \left[1 - 4\left(\frac{\Delta}{\varepsilon}\right)^2 \left(1 - F\left(\frac{2E}{\varepsilon}\right)\right)\right]^2 \tag{8}$$

where $N_0 = m/2\pi d$ is the normal density of states. We assume here that N_0 is structureless in the relevant energy range.

The function F is defined as

$$F(x) = 2x^2 \mathrm{e}^{-x^2} \left[1 + \sum_{n=1}^{\infty} \frac{x^{2n}}{n!(2n+1)}\right].$$

Similarly, integrating equation (7), we obtain

$$\begin{aligned} M^2(E) = \pi^2 N_0^2 &\left[1 - 2\left(\frac{\Delta}{\varepsilon}\right)^2 \left(1 - F\left(\frac{2E}{\varepsilon}\right)\right)\right]^2 4\Delta^2 \\ &\times \int_{-\infty}^{+\infty} \mathrm{d}u\, \rho(u) \left[\frac{2E - u - \mathrm{Re}\,\Sigma}{(2E - u - \mathrm{Re}\,\Sigma)^2 + (\mathrm{Im}\,\Sigma + 2\delta)^2}\right]^2 \end{aligned} \tag{9}$$

where

$$\mathrm{Re}\,\Sigma = \frac{\Delta^2}{2E} F\left(\frac{2E}{\varepsilon}\right) \qquad \mathrm{Im}\,\Sigma = \sqrt{\pi}\,\frac{\Delta^2}{\varepsilon} \mathrm{e}^{-(2E/\varepsilon)^2}.$$

$E - \mathrm{Re}\,\Sigma$ and $\mathrm{Im}\,\Sigma + \delta$ are the real and the imaginary part of the pole of the BPT Green function in the complex ξ plane, respectively.

Recalling the fact that the same coherence factor appears in the matrix elements of both the spin susceptibility and the nuclear spin relaxation rate, we can write the expression of the nuclear spin relaxation rate as follows:

$$\begin{aligned} \frac{T_1^{-1}}{T_{1N}^{-1}} = \int_0^{\infty} \frac{\mathrm{d}E}{2T} \operatorname{sech}^2\left(\frac{E}{2T}\right) \Bigg\{ &\left[1 - 4\left(\frac{\Delta}{\varepsilon}\right)^2 \left(1 - F\left(\frac{2E}{\varepsilon}\right)\right)\right]^2 \\ &+ \left[1 - 2\left(\frac{\Delta}{\varepsilon}\right)^2 \left(1 - F\left(\frac{2E}{\varepsilon}\right)\right)\right]^2 4\Delta^2 \\ &\times \int_{-\infty}^{+\infty} \mathrm{d}u\, \rho(u) \left[\frac{2E - u - \mathrm{Re}\,\Sigma}{(2E - u - \mathrm{Re}\,\Sigma)^2 + (\mathrm{Im}\,\Sigma + 2\delta)^2}\right]^2 \Bigg\} \end{aligned} \tag{10}$$

where T_1^{-1} and T_{1N}^{-1} are the nuclear spin relaxation rate in the superconducting state and in the normal state, respectively. $\delta = 1/2\tau$, where τ is the quasi-particle lifetime.

3. Results and comparison with experiments

3.1. YBaCuO

In order to compare the prediction of the present model with experimental data on YBaCuO, we calculate the variations of T_1^{-1} from equation (10) by assuming $T_c = 90$ K and $v_F = 10^5$ m s^{-1}. The quantities $H_{c_2}(T)$ and $\Delta(T)$ are calculated as indicated in [20]. The curves a, b and c in figure 1 have been obtained in a magnetic field of 20 T. The origin and temperature variation of the quasi-particle lifetime in high-T_c superconductors is still a matter of debate [21]. We consider for this quantity the values and temperature dependence of the particle lifetime commonly derived from experiments. Define $\tau_0 = 0.74\hbar/k_B T_c$ at $T = T_c$. The quasi-particle lifetime is considered as constant and equal to τ_0 in curve a and is assumed in curve b to have the temperature dependence $\tau = \tau_0 T_c/T$ below T_c, which is similar to that of the normal resistance of YBaCuO in the *a*–*b* plane. In curve c, τ is given the temperature variation $\tau = 1.81 \times 10^3 \tau_0 \mathrm{e}^{-7.5T/T_c}$ derived from the experimental study of $\sigma_1(\omega)$ [21]. The emphasis, in the above expressions for τ, is less on the exact form of the variations of this quantity than on the fact that τ^{-1} is equal to constant τ_0 in curve a and exhibits a moderate and a rapid decrease from this value in curve b and c, respectively. The Hebel–Slichter peak is suppressed for constant τ and for a moderate decrease of τ^{-1} below T_c, as shown by curves a and b, while a too steep decrease of τ^{-1} does not lead to the suppression of the coherence peak (curve c). We note, however, that in this case the peak is strongly reduced with regard to the classical low-field BCS behaviour (curve d) calculated for an anisotropy of the energy gap of 0.5 K. Figure 2 shows the temperature variation of T_1^{-1}/T_{1N}^{-1} for different values of the magnetic field H. Our calculation which is valid close to H_{c_2} gives the variations of T_1^{-1} only in the immediate vicinity of T_c and thus does not allow detailed comparison with existing experimental data. However, it shows that as H increases the curves exhibit a decrease in slope and come close to unity.

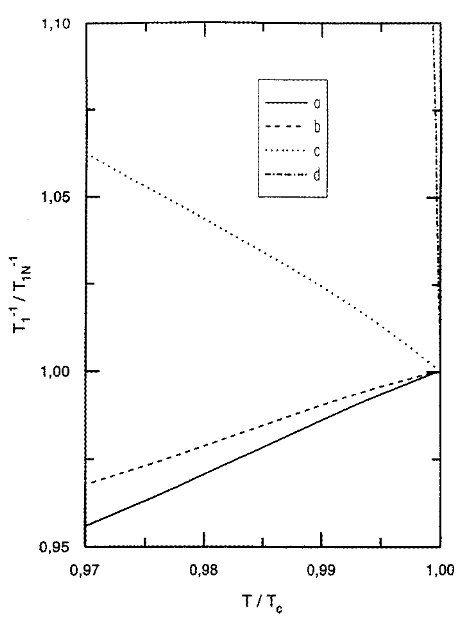

Figure 1. Temperature dependence of the nuclear spin relaxation rate at 20 T for YBaCuO with quasi-particle lifetime τ = constant (curve a), $\tau \sim T^{-1}$ (curve b) and $\tau \sim \mathrm{e}^{-7.5T/T_c}$ (curve c). Curve d shows the classical low-field BCS behaviour.

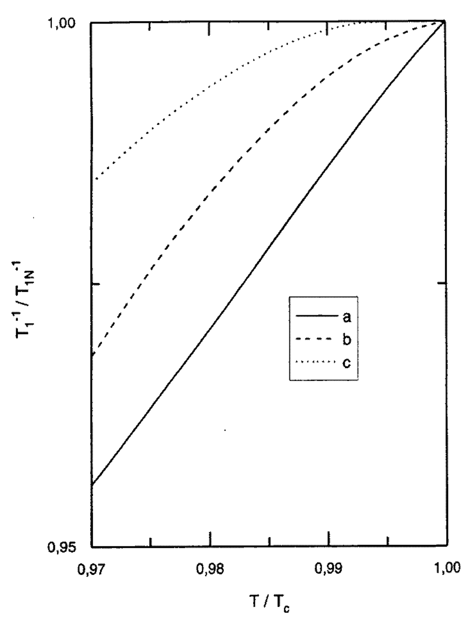

Figure 2. Temperature dependence of the nuclear spin relaxation rate in YBaCuO when quasi-particle lifetime τ is constant and for various magnetic fields: 20 T (curve a); 50 T (curve b) and 70 T (curve c).

3.2. κ-$(ET)_2$ Cu[N(CN)$_2$]Br

Studies of organic superconductors are also characterized by a lack of general consensus as to the nature of the mechanisms of their superconductivity. In particular, an s symmetry order parameter as well as unconventional pairing have been suggested on the basis of experimental data [22]. In figure 3, we report the T_1^{-1}/T_{1N}^{-1} experimental results obtained at magnetic fields 2 T (dots), 3.7 T (squares) and 5.6 T (triangles) by Mayaffre *et al* [11] on organic superconductor κ-$(ET)_2$ Cu[N(CN)$_2$]Br. The curves have been calculated with the following parameter values derived from experiments: $T_c(0) = 12.5$ K; $H_{c_2}(0) \approx 12$ T; $\tau = 0.55\hbar/k_B T_c$. The curves show no coherence peak, and also exhibit a temperature and magnetic field dependence quantitatively consistent with experimental results. We emphasize the fact that this result is obtained with one adjustable parameter, i.e. the value of $T_1 T$ at T_c.

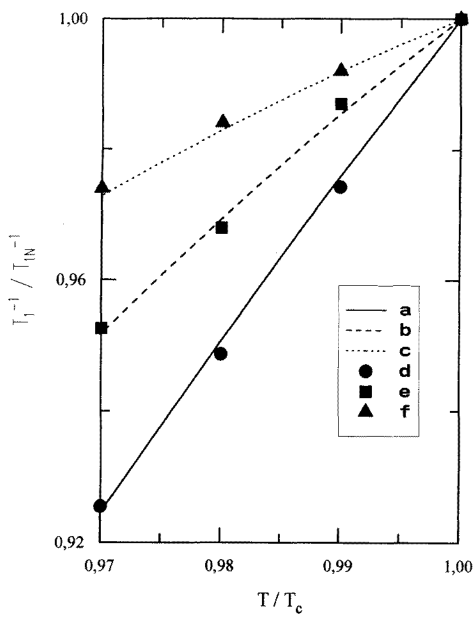

Figure 3. Temperature dependence of the nuclear spin relaxation rate at magnetic fields 2 T (curves a and d), 3.7 T (curves b and e), and 5.6 T (curves c and f) for organic superconductor κ-$(ET)_2$ Cu[N(CN)$_2$]Br. The dots, the squares and the triangles represent the experimental results obtained by Mayaffre *et al* [11].

3.3. Discussion

In 2D and clean s-wave superconductors and weak electron–phonon coupling, the suppression of the coherence peak in strong magnetic fields depends on the magnitude and temperature dependence of the quasi-particle relaxation rate below the transition temperature. The coherence peak is suppressed when the quasi-particle relaxation rate is constant, of the order of $k_B T_c/\hbar$ at $T = T_c$, or when it decreases moderately below T_c from this value. But for a rapid decrease of quasi-particle relaxation rate below T_c, the coherence peak is no longer suppressed. In fact it is incorrect, in this last case, to speak about ‘coherence peak’. The enhancement of the nuclear relaxation rate has the same origin as the enhancement of σ_1 (the real part of the conductivity) which is observed experimentally in YBaCuO, and interpreted on the basis of a drastic suppression of electronic relaxation rate below T_c [21]. The interesting point here is that the enhancement of the nuclear relaxation time is not seen in YBaCuO, while s-wave BCS theory would predict it to occur. In order to decide whether this contradiction is due to the choice of s-wave symmetry, one should derive the theory for the d-wave symmetry in the clean limit and for large fields [23]. In any case, the theory framework, as it stands in this paper is clearly unable to describe the physics of YBaCuO.

On the other hand, the success of the theory in the case of the quasi-2D organic superconductor mentioned in the previous section indicates that the model does incorporate some of the essential physics. Obviously, a significant difference in the microscopic physics of the organic superconductor as compared to YBaCuO is that the electronic relaxation time in the latter is practically independent of the temperature in all of the superconducting phase; one consequence, if this is correct, is that the microwave conductivity σ_1 should not behave like that of YBaCuO below T_c and should not exhibit the striking deviations from BCS theory which are found in the latter compound. Indeed, microwave measurements of $\sigma_1(\omega)$ in organic superconductors show a rapid decrease of σ_1 below T_c, as expected [24].

Both the suppression of the coherence peak and the magnetic field dependence of T_1^{-1} arise from the effect of the magnetic field on the density of states. The density of states $N(\omega, \theta)$, calculated by BPT for clean 3D superconductors in strong magnetic fields, appears to vary continuously from the normal density of state N_0 to the BCS curve when the parameter $\Delta / k v_F \sin\theta$ runs from zero to infinity. Therefore, for sufficiently large magnetic fields the divergence at $\omega = \Delta$ in the density of states is wiped out and, consequently, a significant decrease and even the suppression of the coherence peak in the nuclear spin relaxation rate occurs since this quantity depends on the average over ω of the square of the density of states. This is even more pronounced in 2D systems where $\sin\theta$ reaches its maximum value equal to unity. As the magnetic field becomes larger, the density of states tends towards the normal density of states N_0 making the T_1^{-1}/T_{1N}^{-1} curves come close to unity.

The agreement of our microscopic calculation with experiments in the case of the organic superconductor is also surprising, at first sight, in view of the fact that the vortex system close to H_{c_2} in this strongly anisotropic system is in a liquid state [18]. Our calculation assumes a periodic Abrikosov array of vortices. In fact, it shows that in this part of the phase diagram, where the London penetration depth is much larger than the intervortex mean distance, the vortex thermal fluctuations do not influence the nuclear relaxation rate. At a larger distance from the $H_{c_2}(T)$ mean field curve, when the penetration depth has decreased significantly, and the system is close to a solid–liquid transition, it is likely that vortex thermal fluctuations have a noticeable effect on $T_1 T$. It would be useful to extend the validity of our treatment to such regions of the phase diagram, but this requires more work, since our perturbation theory approach is valid only in the vicinity of H_{c_2}. In the absence of such a complete theory we can nevertheless state that close to H_{c_2} the behaviour of the nuclear relaxation time is governed by the order parameter fluctuations and not by the vortex thermal fluctuations.

Within the theoretical framework of the present paper, one should have a coherence peak in the low-field limit since the density of states is then of BCS type, whereas experimental T_1^{-1} shows no coherence peak in weak magnetic fields. Also, for a magnetic field H, which makes an angle ϕ with the c-axis, only the z-component of the magnetic field H, i.e. $Hx\cos\phi$, intervenes in the present calculation of the nuclear spin relaxation rate and when H is parallel to the superconducting planes there is no orbital effect. The density of states is of the BCS type and so we expect from the present model that the coherence peak is restored as the magnetic field is rotated from a direction perpendicular to the planes to a direction parallel to the planes in contradiction with experiments. These results suggest that the suppression of the Hebel–Slichter peak in real systems may be linked with other factors such as the non-s-symmetry of the order parameter and strong electronic interactions [12], which the model does not take into account, and that the BCS nuclear spin relaxation rate behaviour is recovered at least in the vicinity of T_c, in the high-field regime, when the magnetic field is perpendicular to the superconducting planes.

4. Conclusion

In this paper, we have investigated the NMR relaxation time T_1 expected in an s-wave, two-dimensional BCS superconductor in the clean limit. To our knowledge, this had not been derived previously. We have found, not too surprisingly after second thoughts, that the Hebel–Slichter anomaly is suppressed in strong fields. This results from the strong suppression at large fields of the density of states square-root singularity responsible for the Hebel–Slichter anomaly at low fields. A specific feature of the result is the expected restoration of the Hebel–Slichter anomaly in a parallel field.

Various reasons exist as to why our results do not account for experimental findings in superconducting cuprates. The weak-coupling BCS approach takes no account of the strong electronic interactions known to be at work in those systems. The spectrum of low-lying spin fluctuations is likely to be strongly affected by such electronic correlations included in the vicinity of the normal phase. Another possible reason is the symmetry of the order parameter, which may be d-wave in superconducting cuprates [25] and possibly not s-wave in organics [22, 11]. It should be remembered that in a quasi-2D superconductor the thermal fluctuations of the vortex lattice are expected to contribute to the NMR relaxation rate, at least in the neighbourhood of a vortex lattice melting transition, an effect which is completely absent in our calculation. Our results suggest that close to H_{c_2}, the variations of T_1^{-1} are governed by the order parameter fluctuations and not by vortex thermal fluctuations. Experiments conducted in the flux flow regime, however, contain the effect of the fluctuating vortex liquid. The NMR relaxation time in a quasi-2D d-wave BCS superconductor in large fields in the clean limit is currently being investigated and should be reported in the near future [23].

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Appendix

The integral equation for a pure type II superconductor Green function near the upper critical field is written as

$$G_\omega(r, r') = G^0_\omega(r - r') - \int \mathrm{d}^3r_1\, \mathrm{d}^3r_2\, G^0_\omega(r - r_1) V(r_1, r_2) G^0_{-\omega}(r_1 - r_2) G_\omega(r_2, r') \tag{A1}$$

where G^0_ω is the Green function of the normal metal in the absence of the magnetic field. The potential

$$V(r_1, r_2) = \Delta^*(r_1)\Delta^*(r_2) \exp[-\mathrm{i}eH(x_1 + x_2)(y_1 + y_2)] \tag{A2}$$

describes a superconductor in the mixed state in a magnetic field whose spatial average is H.

For the order parameter $\Delta(r)$, BPT consider the form of the Abrikosov vortex solution derived from Ginzburg–Landau theory for conventional superconductors:

$$\Delta(r) = \sum_n C_n \mathrm{e}^{\mathrm{i}nqy} \exp\left[-eH\left(x - \frac{qn}{2eH}\right)^2\right]. \tag{A3}$$

This form is strictly correct only in the Ginzburg–Landau region ($T \sim T_c$), but remains qualitatively correct far from T_c.

Since $G_\omega(r, r')$ depends on the sum coordinates, $G_\omega(r, r')$ and its Fourier components, $G_\omega(p, k)$ have the periodicity of the flux-line lattice. $G_\omega(p, k)$ is given by the following equation:

$$G_\omega(r, r') = \sum_k \exp\left[\mathrm{i}\frac{1}{2}k(r + r')\right] \int \frac{\mathrm{d}^3 p}{(2\pi)^3} G_\omega(p, k) \exp[\mathrm{i}p(r - r')] \qquad \text{(A4)}$$

where the momentum k takes on all the discrete values of a two-dimensional lattice reciprocal to the spatial lattice of flux lines whose period is $(\hbar/2eH)^{1/2}$.

Equation (A1) can be written in terms of the Fourier coefficients $G_\omega(p, k)$ as follows for fixed p:

$$\begin{aligned} G_\omega\left(p - \frac{k}{2}, -k\right) &= \delta_{k,0}G^0_\omega(p) - G^0_\omega(p - k)\sum_{k'} G_\omega\left(p - \frac{k + k'}{2}, -k - k'\right) \\ &\quad \times \int \frac{\mathrm{d}^3 p'}{(2\pi)^3} V(p', k') G^0_{-\omega}\left(p - p' - k - \frac{k'}{2}\right) \qquad \text{(A5)} \end{aligned}$$

where $V(p, k)$ is the Fourier component of $V(r, r')$.

In order to determine $G_\omega(p, 0)$, BPT consider an Abrikosov square flux-line lattice and, by neglecting the Fourier coefficients Δ^2_k of the absolute square of the order parameter for $k \neq 0$ with regard to Δ^2_0, they derived from equation (A5) the expression given in (4).

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